Roll No : FA18-BSE-048

$a = 0$, $b = 4$, $c = 8$

Dataset

x	y
1	$4-(0)(1)^4-(4)(1)^2+8(1) = 8$
1.25	$7.00391-0-(4)(1.25)^2+8(1.25)$
1.5	$12.5625-0-4(1.5)^2+8(1.25)$
1.75	$21.56641-0-4(1.75)^2+8(1.75)$
2	$35-0-4(2)+8(2)$

Difference Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	8				
1.25	10.75391	2.75391			
			2.05468		
1.5	15.5625	4.80859		0.89864	
			2.94532		0.09372
1.75	23.31641	7.75391		0.98436	
			3.92968		
2	35	11.68359			

Interpolate $x = 1.2$ using Forward Diff Method

$$h = 1.25 - 1 = 0.25$$

$$p = \frac{x-1}{h} = \frac{1.2-1}{0.25} = 0.8$$

$$P(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$P(1.2) = 8 + 0.8(2.75391) + \frac{0.8(0.8-1)}{2}(0.05468) + \frac{0.8(0.8-1)(0.8-2)}{6}(0.89064) + \frac{(0.8)(0.8-1)(0.8-2)(0.8-3)}{24}(0.09372)$$

$$\boxed{P(1.2) = 10.0656}$$

Interpolate $x = 1.6$ using Stirling Diff Method

$$p = \frac{x - 1.5}{h} = \frac{1.6 - 1.5}{0.25} = 0.4$$

~~$\Delta y_0 = (4.8086 + 7.7539)/2 = 7.7539$~~

~~$\Delta^3 y_0 = (0.8906 + 0.9844)/2 = 0.9844$~~

$$P(x) = y_0 + p\mu\delta y_0 + \frac{p^2\delta^2 y_0}{2!} + \frac{p(p^2-1)}{3!}\mu\delta^3 y_0 + \frac{p^2(p^2-1)}{4!}\delta^4 y_0$$

$$= 15.5625 + (0.4)\frac{(7.7539 + 4.8086)}{2} + \cdots$$

$$+ \frac{(0.4)(0.4-1)}{2}(0.9844 + 0.8906)$$

$P(1.6) = 1.525 + (0.4)\left(\frac{7.9539 + 4.8086}{2}\right) + \frac{0.16}{2}(2.9453)$

$+ \frac{(0.4)(0.16-1)}{6}(0.0937) + \frac{(0.4)(0.16-1)}{2}(0.9844 + 0.8906) + \frac{(0.16)(0.16-1)}{24}$

midpoints

$\boxed{P(1.6) = 18.2576}$

interpolating $x = 1.9$ using backward Diff method

$$p = \frac{1.9 - 2}{0.25} = -0.4$$

$$P(x) = y_0 + p\nabla y_0 + \frac{p(p+1)}{2!}\nabla^2 y_0 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_0 + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_0$$

$$P(1.9) = 35 + (-0.4)(11.68359) + \frac{(-0.4)(-0.4+1)}{2}(3.92968) + \frac{(-0.4)(-0.4+1)(-0.4+2)}{6}(0.98436) + \frac{(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)}{24}(0.09372)$$

$\boxed{P(1.9) = 29.7881}$